**Chapter 4**

**Differentiation of Functions of Several Variables**

**4.3 Partial Derivatives**

**Section Exercises**

**For the following exercises, calculate the partial derivative using the limit definitions only.**

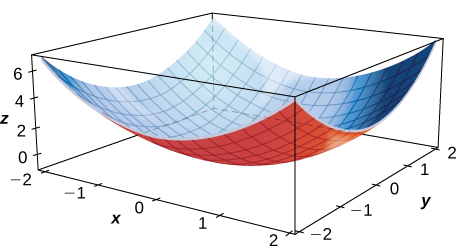
1.  for 

Answer: 

1.  for 

Answer: 

**For the following exercises, calculate the sign of the partial derivative using the graph of the surface.**



1. 

Answer: The sign is positive.

1. 

Answer: The sign is negative.

1. 

Answer: The sign is positive.

1. 

Answer: The partial derivative is zero at the origin.

**For the following exercises, calculate the partial derivatives**.

1.  for 

Answer: 

1.  for 

Answer: 

1.  and  for 

Answer: 

1.  and  for 

Answer: 

1. Find  for 

Answer: =

1. Let  Find  and 

Answer: 

1. Let  Find  and .

Answer: 

1. Let  Find  and 

Answer: 

1. Let  Find  and 

Answer 

1. Let  Evaluate  and 

Answer: 

1. Let  Find and 

Answer: 

**Evaluate the partial derivatives at point .**

1. Find  at  for 

Answer: 

1. Given  find  and 

Answer: , 

1. Given  find  and 

Answer: 

1. The area of a parallelogram with adjacent side lengths that are, and in which the angle between these two sides is *,* is given by the function . Find the rate of change of the area of the parallelogram with respect to the following:
2. Side *a*
3. Side *b*
4. 

Answer: a.  b.  c. 

1. Express the volume of a right circular cylinder as a function of two variables:
2. its radius and its height .
3. Show that the rate of change of the volume of the cylinder with respect to its radius is the product of its circumference multiplied by its height.
4. Show that the rate of change of the volume of the cylinder with respect to its height is equal to the area of the circular base.

Answer: a.  b.  c. 

1. Calculate for 

Answer: 

**Find the indicated higher-order partial derivatives.**

1.  for 

Answer: 

1.  for 

Answer: 

1. Let. Find  and .

Answer: 

1. Given , find  and 

Answer: ==

1. Given , find  and 

Answer: 

1. Given , show that 

Answer: =

1. Show that  is a solution of the differential equation .

Answer: 

1. Find  for 

Answer: 

1. Let . Find .

Answer: 

1. Let  Find .

Answer:

1. Given , find all points at which  simultaneously.

Answer: 

1. Given , find all points at which  and  simultaneously.

Answer: .

1. Given , find all points on  at which  simultaneously.

Answer: 

1. Given , find all points at which  simultaneously.

Answer: 

1. Show that  satisfies the equation .

Answer: 

1. Show that  solves Laplace’s equation 

Answer: 

1. Show that  satisfies the heat equation .

Answer: 

1. Find  for .

Answer: 

1. Find  for .

Answer: 

1. Find for .

Answer: 

1. Find  for .

Answer: 

1. The function  gives the pressure at a point in a gas as a function of temperature  and volume . The letters  are constants. Find  and  and explain what these quantities represent.

Answer: . These quantities represent the rate of change of pressure with respect to volume and the rate of change of pressure with respect to temperature.

1. The equation for heat flow in the -plane is . Show that  is a solution.

Answer: This is a proof; therefore, no answer is provided.

1. The basic wave equationis  Verify that and  are solutions.

Answer: This is a proof; therefore, no answer is provided.

1. The law of cosines can be thought of as a function of three variables. Let  and  be two sides of any triangle where the angle  is the included angle between the two sides. Then,  gives the square of the third side of the triangle. Find  and  when  and 

Answer: 

1. Suppose the sides of a rectangle are changing with respect to time. The first side is changing at a rate of  in./sec whereas the second side is changing at the rate of  in/sec. How fast is the diagonal of the rectangle changing when the first side measures  in. and the second side measures  in.? (Round answer to three decimal places.)

Answer:  in./sec

1. A Cobb-Douglas production function is  where  represent the amount of labor and capital available. Let  and Find  and  at these values, which represent the marginal productivity of labor and capital, respectively.

Answer: at  at

1. The apparent temperature index is a measure of how the temperature feels, and it is based on two variables:  which is relative humidity, and  which is the air temperature. . Find  and  when  and .

Answer:  at,  at

**Student Project**

**Lord Kelvin and the Age of the Earth**

1. Substitute this form into  and, noting that  is constant with respect to distance  and  is constant with respect to time, show that



Answer: This is a proof; therefore, no answer is provided.

1. This equation represents the separation of variables we want. The left-hand side is only a function of  and the right-hand side is only a function of , and they must be equal for all values of . Therefore, they both must be equal to a constant. Let’s call that constant  (The convenience of this choice is seen on substitution.) So, we have

Now, we can verify through direct substitution for each equation that the solutions are  and  where  Note that is also a valid solution, so we could have chosen  for our constant. Can you see why it would not be valid for this case as time increases?

Answer: This is a proof; therefore, no answer is provided.

1. Let’s now apply boundary conditions.
2. The temperature must be finite at the center of Earth,  Which of the two constants,  or , must therefore be zero to keep finite at? (Recall that  as  but  behaves very differently.)
3. Kelvin argued that when magma reaches Earth’s surface, it cools very rapidly. A person can often touch the surface within weeks of the flow. Therefore, the surface reached a moderate temperature very early and remained nearly constant at a surface temperature. For simplicity, let’s set and find  such that this is the temperature there for all time . (Kelvin took the value to be  We can add this  constant to our solution later.) For this to be true, the sine argument must be zero at . Note that  has an infinite series of values that satisfies this condition. Each value of  represents a valid solution (each with its own value for). The total or general solution is the sum of all these solutions.
4. At , we assume that all of Earth was at an initial hot temperature  (Kelvin took this to be about.) The application of this boundary condition involves the more advanced application of Fourier coefficients. As noted in part b. each value of  represents a valid solution, and the general solution is a sum of all these solutions. This results in a series solution:

Answer: This is a proof; therefore, no answer is provided.

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